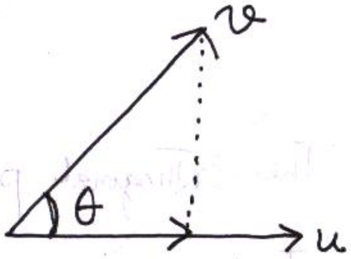
Let V be an inner product space.

Let u & v are two non-zero vectors such that θ be the angle between them.

Then, the orthogonal projection of v along the vector u, denoted by $\mathrm{Proj}_u(v)$ is obtained as follows:



$$\therefore\ \mathrm{Proj}_u(v) = \|v\| \cos\theta \left(\frac{u}{\|u\|}\right)$$

$$= \frac{\|v\| \cdot \left\langle \frac{u}{\|u\|}, v \right\rangle}{\left\| \frac{u}{\|u\|} \right\| \|v\|} \frac{u}{\|u\|}$$

[$\because$ for $u \neq 0, v \neq 0$ in an i.p. space, there exist a unique $\theta \in [0, \pi]$ s.t. $\cos\theta = \frac{\langle u, v\rangle}{\|u\| \|v\|}$]

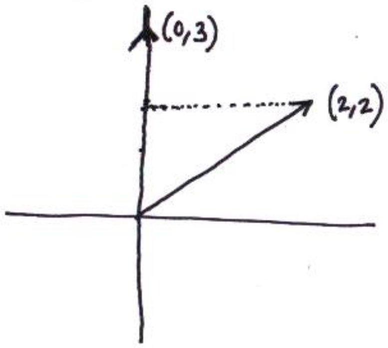
$$= \|v\| \frac{\langle u, v\rangle}{\|u\| \|v\|} \frac{u}{\|u\|}$$

[$\because$ for $u \neq 0$, $\left\|\frac{u}{\|u\|}\right\| = 1$] & $\langle \alpha u, v\rangle = \alpha \langle u, v\rangle$ for $\alpha \in \mathbb{R}$.

$$= \frac{\langle u, v\rangle}{\|u\|^2} u$$

[$\because$ for $v \neq 0$, $\|v\| \neq 0$]

Ex:- Find the orthogonal projection of $v = (2,2)$ along the vector $u = (0,3)$ in $\mathbb{R}^2$.



$$\mathrm{Proj}_u(v) = \frac{u \cdot v}{\|u\|^2} u = \frac{(0,3) \cdot (2,2)}{9} (0,3)$$

$$= \frac{6}{9} (0,3)$$

$$= (0,2).$$