Indian Institute of Space Science and Technology

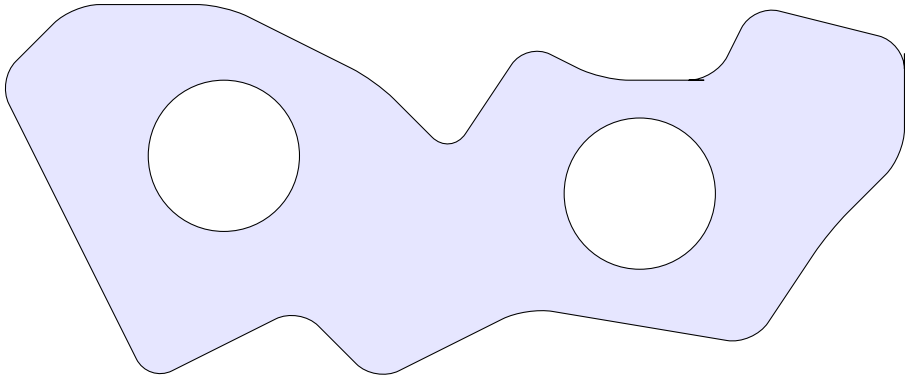
MA121-Vector Calculus Assignment, 2014

1. Let C be a smooth curve and $f : C \longrightarrow \mathbb{R}^3$ be a continuous scalar field. Show that
$$\int_C f = \int_{-C} f$$

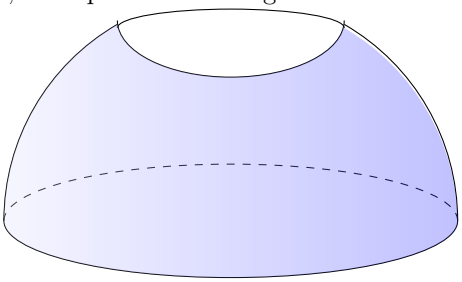
2. Let C be a smooth curve and $\vec{F} : C \longrightarrow \mathbb{R}^3$ be a continuous vector field. Show that
$$\int_C \vec{F} = -\int_{-C} \vec{F}$$

3. Using green's theorem prove the 2nd structure theorem (using curl) of conservative vector field.

4. Let $\vec{F}$: $\mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a smooth vector field and let G be a region in XY-plane with positively oriented smooth boundaries C_1, C_2 and C_3 as depicted in the figure. Derive Green's for the vector field $\vec{F}$ over the region G.



5. Let $\vec{F}$= (xy, yz, zx) be a vector field defined over $\mathbb{R}^3$. Let S be the part of upper hemisphere with radius 5 and center $(0, 0, 0)$ between the plane $z = 0$ and $z = 3$; as depicted in the figure below. Find the surface integral of Curl$(\vec{F})$ over the surface S using Stoke's theorem.



6. Let $\vec{F}$ be a smooth vector field with domain $\mathbb{R}^2 - \{(0, 0)\}$ having Curl$(\vec{F}) = 0$. Let C_1 and C_2 be any two positively oriented smooth loops around the point as depicted in the figure. Show that line integrals of $\vec{F}$ along C_1 and along C_2 are the same.