Quantum communication, Information, and Sensing

Precision measurements by non-Gaussian operators

K. Amrutha[1], C. Sudheesh[1]

Department of physics, Indian Institute of Space science and Technology,

Valiamala, Thiruvananthapuram, 695547

ABSTRACT

In this study, we theoretically investigate that, for a given coherent state amplitude α and a given squeezing parameter r, the mixing of coherent states and photon-added squeezed vacuum states(PA-SVS) at the first beam splitter of an interferometer leads to improved phase-shift measurement sensitivity when using the photon-number detection technique on one of the output beams of the device. Results show that photon-added squeezed vacuum state can give the better phase sensitivity and resolution than both the squeezed vacuum state(SVS) and the photon-subtracted squeezed vacuum state(PS-SVS) of the same field parameters.

INTRODUCTION

- Optical interferometry, particularly the Mach-Zehnder Interferometer (MZI), is widely used in quantum precision measurements, enabling applications in gravitational wave detection, optical gyroscopes, and environmental monitoring.
- The ultimate goal of quantum-enhanced interferometry is to estimate an unknown phase ϕ beyond the Standard Quantum Limit (SQL) or Shot-Noise Limit (SNL).
- It is mentioned in the previous work that non-Gaussian operators enhance the nonclassical properties of state, so such states can be used for sensing purpose and enhance sensitivity. We considering photon addition operators for this study

METHODOLOGY

We consider a Mach-Zehnder interferometer, with input coherent and p-photon added squeezed vacuum states, denoted $|\alpha\rangle_a$ and $|r,p\rangle_b$ respectively, input state to the MZI in the angular momentum basics $|j,m\rangle$ is

$$|in\rangle = |\alpha\rangle_a \otimes |r,p\rangle_b$$

$$= \sum_{j=\frac{P}{2},\frac{P+1}{2},\frac{p+2}{2},\ldots}^{\infty} \sum_{m=-j}^{j-P} G_{j,m,p}|j,m\rangle$$

Where, $G_{j,m,P} = A_{j+m}\, B_{j-m-P}$

Output state of MZI, $|out\rangle = e^{-i\phi\widehat{J_2}}|in\rangle$

Expectation value of parity operator with respect to output state $|out\rangle$,

$$\langle\widehat{\Pi}\rangle = \langle in|e^{i\phi\widehat{J_2}}e^{i\pi(\widehat{J_0}-\widehat{J_3})}e^{-i\phi\widehat{J_2}}|in\rangle$$

$$= \sum_{j=\frac{P}{2},\frac{1+P}{2},\frac{2+P}{2},\ldots}^{\infty} \sum_{m=-j}^{j-P}\sum_{m'=-j}^{j-P}\sum_{m''=-j}^{j-P}\left((-1)^{(j-m'')}\right) G_{j,m,P}\; G^{*}_{j,m',P}\; d^{j}_{m'',m}(\phi)\, d^{j}_{m',m''}(-\phi)$$

For the measurement of photon number parity, the phase uncertainty based on the error propagation calculus is given by

$$\Delta\phi = \frac{\Delta\widehat{\Pi}_b}{\left|\frac{\partial\langle\widehat{\Pi}\rangle}{\partial\phi}\right|}, \qquad \text{where } \Delta\widehat{\Pi}_b = \sqrt{1-\langle\widehat{\Pi}_b(\phi)\rangle^2}$$

CONCLUSION

- We found that for given state parameters α and r, improvement in the sensitivity (reduction in the noise of the phase- shift measurement) occurs generally with the increasing number of photon added.
- Increasing resolution with increasing numbers of photons added

REFERENCES

1. R. Birrittella and C. C. Gerry, J. Opt. Soc. Am. B, 31 (2014) 586.
2. S. Wang, X. X. Xu, Y. J. Xu, et al., Opt. Commun., 444 (2019) 102
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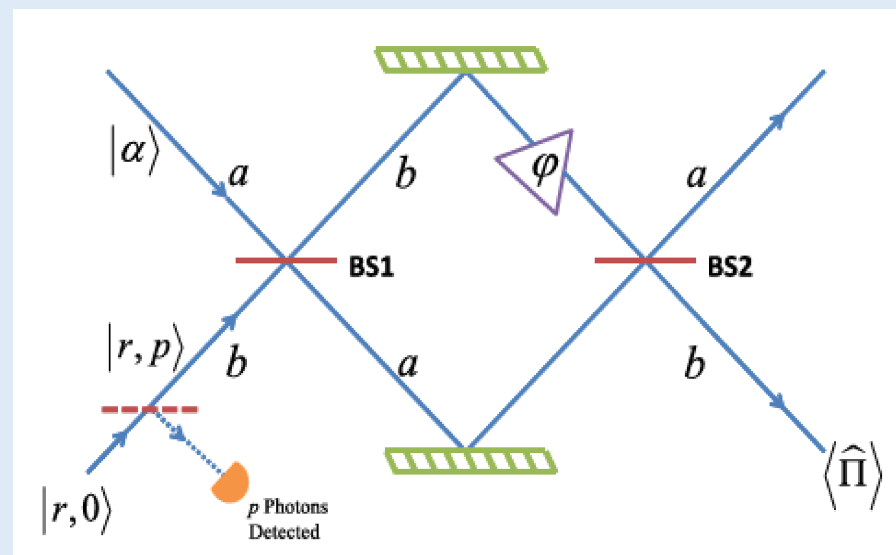
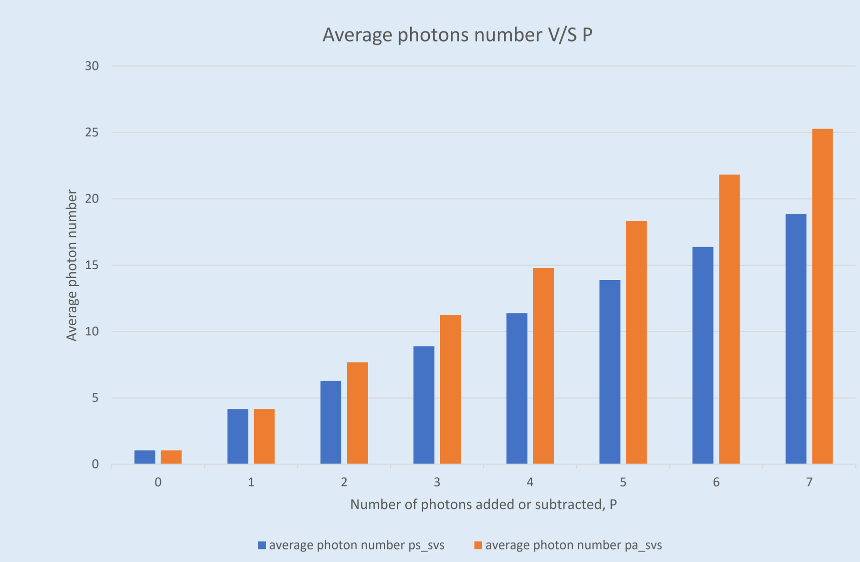
SYSTEM

Mach–Zehnder interferometer (MZI)





Input states : Coherent state ($|\alpha\rangle$) and Photon added squeezed vacuum state ($|r,p\rangle$)
Detector : Photon number parity detector

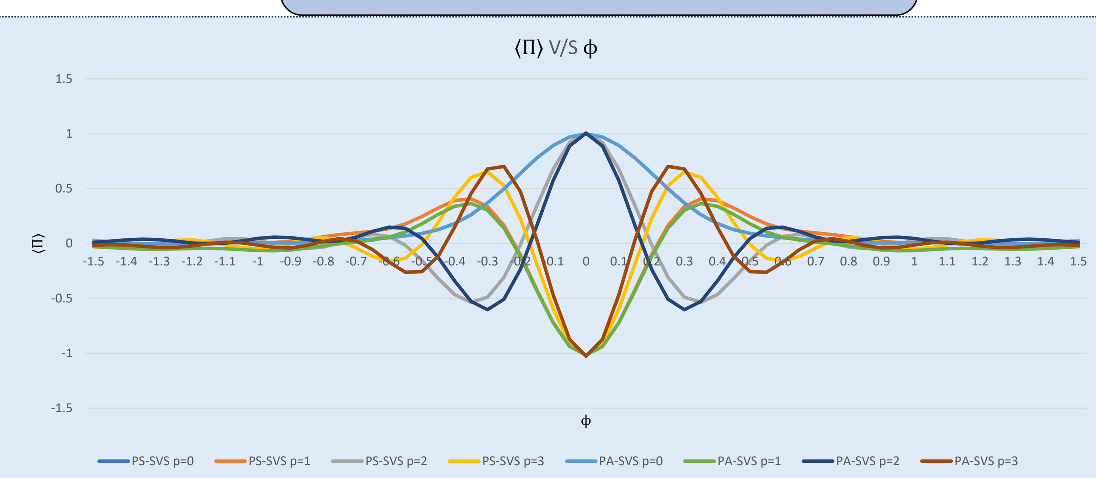
Coherent state, $|\alpha\rangle = \sum_{n=0}^{\infty} A_n|n\rangle,\; A_n = e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^n}{\sqrt{n!}}$

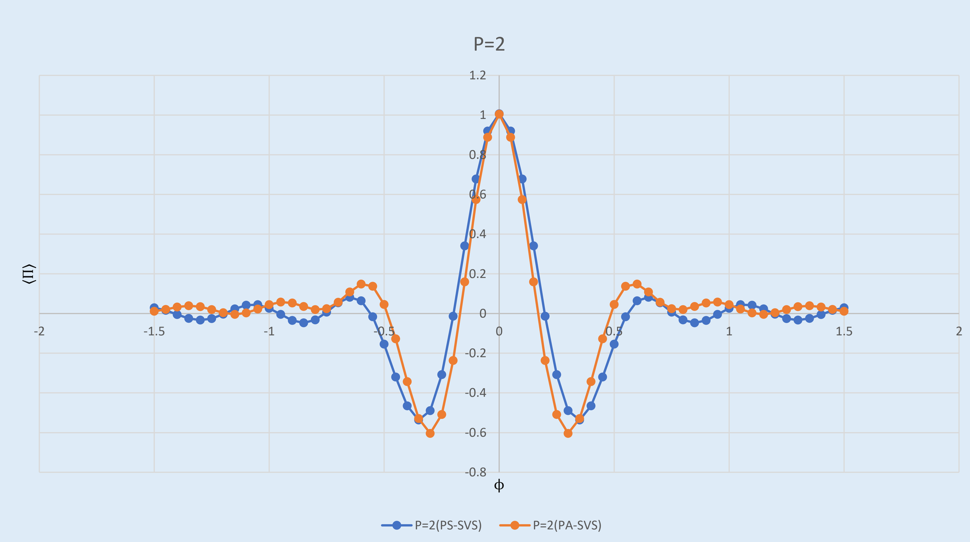
P photon added SVS, $|r,P\rangle_b = (b^{+})^{P}|r,0\rangle_b = \sum_{k=0}^{\infty} B_k^{(P)}|k+P\rangle$

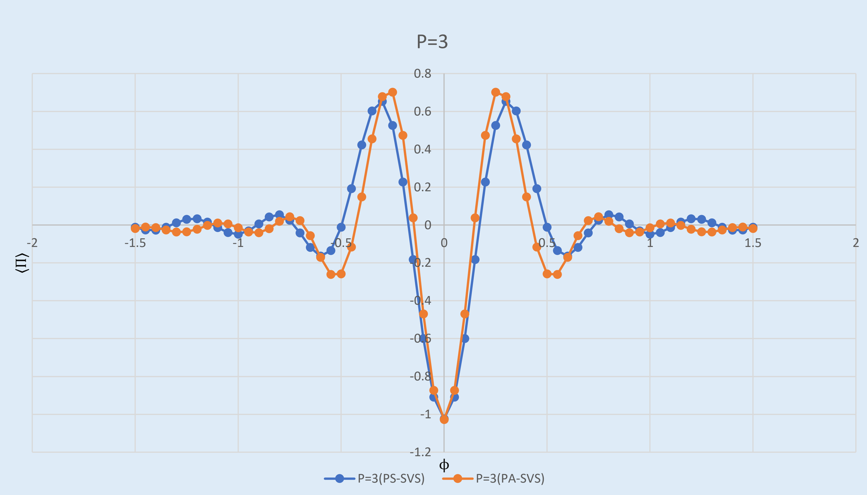
$$B_k^{(P)} = N_P C_k\left[\sqrt{\frac{(k+P)!}{k!}}\right], \qquad N_P = \left(\sum_{l=0}^{\infty}|C_l|^2\left[\frac{(l+P)!}{l!}\right]\right)^{-\frac{1}{2}}$$

$$C_l = (-1)^{\frac{l}{2}}\left[\frac{l!}{2^l\left(\frac{l}{2}!\right)^2}\frac{tanh^l r}{coshr}\right]^{\frac{1}{2}} cos^2\left(\frac{l\pi}{2}\right)$$

RESULT







- In quantum metrology, average photon number of input states is an important factor. **PA-SVS have better average number of photon than PS-SVS and SVS**
- As more photons are subtracted, $\langle\widehat{\Pi}(\phi)\rangle$ vs ϕ **become sharper around ϕ =0**

=> Narrow fringes = better phase resolution: the interferometer can distinguish smaller phase differences.

- Since $\langle\widehat{\Pi}\rangle$ changes more rapidly with ϕ as p increases (steeper slope near ϕ = 0), the derivative $\frac{\partial\langle\widehat{\Pi}\rangle}{\partial\phi}$ becomes larger.

=> This implies **lower phase uncertainty** via error propagation formula, So, photon subtraction leads to **more sensitive measurements**.

- **Input states** : Coherent state ($|\alpha\rangle$) and Photon added squeezed vacuum state ($|r, p\rangle$)
- **Detector** : Photon number parity detector