Pulse Radar:

MTI

Lecture 24-27

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Introduction

Radars

- CW
  - FMCW
- Pulsed
  - Noncoherent
  - Coherent
    - Low PRF
    - Medium PRF
      - MTI
      - Pulse Doppler
    - High PRF

Note:
- CW = continuous wave
- FMCW = frequency modulated continuous wave
- PRF = pulse repetition frequency
- MTI = moving target indicator
PRT

Carrier Freq.

PRT = 1/PRF

PW
Magnitude of the amplitude spectrum for a finite number of rectangular pulses \([(N + 1) = 5\]
Pulse Effects on System Performance

Pulse Shape

Pulse Width

Pulse Compression

Pulse power
Pulsed Radar Parameters

- Pulsed radar waveforms can be completely defined by the following:
  - (1) carrier frequency which may vary depending on the design requirements and radar mission.
  - (2) pulse width, which is closely related to the bandwidth and defines the range resolution;
  - (3) modulation; and
  - (4) the pulse repetition frequency
Pulsed Radar Parameters

\[ P_0 \]

\[ T_p \]

\[ r \]

Combined returned from target 1 and 2

Transmit pulse

\[ R_1 = \frac{ct_1}{2} \]

\[ R_2 = \frac{ct_2}{2} \]

\[ \Delta R \]
<table>
<thead>
<tr>
<th>PRF</th>
<th>Range Ambiguous</th>
<th>Doppler Ambiguous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low PRF</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Medium PRF</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>High PRF</td>
<td>Yes</td>
<td>No</td>
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</tbody>
</table>
MTI Radar

- The purpose of moving-target indication (MTI) radar is to reject signals from fixed or slow-moving unwanted targets, such as buildings, hills, trees, sea, and rain, and retain for detection or display signals from moving targets such as aircraft.

Simplified Block diagram of a MTI system
• Moving target indication (MTI) is a mode of operation of a radar to discriminate a target against clutter.
• The MTI radar uses Low Pulse Repetition Frequency (PRF) to avoid range ambiguities.
• Radar MTI may be specialized in terms of the type of clutter and environment:
  • Airborne MTI (AMTI),
  • Ground MTI (GMTI), etc., or
  • may be combined mode: stationary and moving target indication (SMTI).
\[ V_{\text{diff}} = A_d \sin \left[ 2\pi f_d t - \frac{4\pi f_t R_0}{c} \right] \]
Radar Signal

\[ T_{PRI} \]

\[ T \]

\[ T_{CPI} = N T_{PRI} \]

\[ Time \]

\[ T = \text{Pulse Length} \]
\[ B = \frac{1}{T} = \text{Bandwidth} \]

\[ T_{PRI} = \] Pulse Repetition Interval (PRI)
\[ f_P = \frac{1}{T_{PRI}} = \text{Pulse Repetition Frequency (PRF)} \]
\[ \delta = \frac{T}{T_{PRI}} = \text{Duty Cycle (\%)} \]

\[ T_{CPI} = N T_{PRI} = \] Coherent Processing Interval (CPI)
\[ N = \] Number of pulses in the CPI

\[ N = 2, 3, \text{ or } 4 \text{ for MTI} \]
\[ N \text{ usually much greater (8 to } \sim 1000) \text{ for Pulse Doppler} \]
(a) echo pulse train;

(b) video pulse train for doppler frequency $f_d > l/\tau$;
• Moving targets distinguished from stationary targets by observing the video output on an A-scope (amplitude vs. time).

(a-e) Successive sweeps of an MTI radar A-scope display (echo amplitude as a function of time); (f) superposition of many sweeps; arrows indicate position of moving targets
The simple MTI delay-line canceler is used to reject stationary clutter at zero frequency.
Superposition of many sweeps; arrows indicate position of moving targets

\[ V_{\text{output}} = V_i - V_{i-1} \]
Effect of delay line canceller on the signal
Delay Line Canceler

- The video signal received from a particular target at a range $R_0$ is
  \[ V_1 = k \sin(2\pi f_d t - \phi_0) \]
  Where $\phi_0$ = phase shift and $k$ = amplitude of video signal.

- The signal from the previous transmission, which is delayed by a time $T = \text{pulse repetition interval}$, is
  \[ V_2 = k \sin[2\pi f_d (t - T) - \phi_0] \]

- The output of the subtractor is
  \[ V_1 - V_2 = 2k \sin \pi f_d T \cos \left[ 2\pi f_d \left( t - \frac{T}{2} \right) - \phi_0 \right] \]
The magnitude of the relative frequency-response of the delay-line canceler

\[ H(f) = \frac{2k \sin(\pi f_d T)}{k} = 2 \sin(\pi f_d T) \]

Frequency response of the single delay-line canceler; \( T \) = delay time
• Properties of DLC

• The frequency response function has zero response when moving targets have doppler frequencies at the prf and its harmonics.

• The clutter spectrum at zero frequency is not a delta function of zero width but has a finite width so that clutter will appear in the pass band of the DLC.
Blind Speed

\[ v_n = n\lambda f_r \approx n\lambda f_r \]

- Where, \( v_n \) is the nth blind speed in knot
- \( \lambda \) operating wavelength in m
- \( f_r \) is prf in Hz

Plot of MTI radar first blind speed as a function of \( R_{un} \).
**Limitation in MTI Radar**

- The blind speeds are **one of the limitations of pulse MTI radar** which do not occur with CW radar.
- Blind speed can be a serious limitation in MTI radar since they cause some desired moving targets to be cancelled along with the undesired clutter at zero frequency.

\[ v_n = \frac{n\lambda f_r}{2} = \frac{n\lambda}{2T} \quad n = 1,2,3,... \]

- Based on the equation, there are four methods for reducing the detrimental effects of blind speeds.
  - Operate the radar at lower frequencies [or long wavelengths]
  - Operate with a high prf
  - Operate with more than one prf [known as staggered-prf MTI]
  - Operate with more than one RF frequency [or wavelength]
Clutter Spectrum

Two Pulse MTI Canceller Frequency Response

Aliased Clutter Spectra

Aliased Two Pulse MTI Response

Relative MTI Filter Response

Frequency

$F_{PRF}$

$2F_{PRF}$

Relative response

Frequency

$1/F_2$, $2/F_2$, $3/F_2$, $4/F_2$, $5/F_2$

Relative response

Frequency

$1/F_2$, $1/F_1$, $2/F_2$, $2/F_1$, $3/F_2$, $3/F_1$, $4/F_2$, $4/F_1$, $5/F_2$
Clutter Spectrum

- Land
- Sea
- Rain
- Chaff
- Birds

Relative Power (dB)

Velocity (m/s)

Target
N Pulse Delay line Canceler

- The frequency response of a single-delay-line canceler does not always have as broad a clutter-rejection null as might be desired in the vicinity of d-c.
- The clutter-rejection notches may be widened by passing the output of the delay-line canceler through a second delay-line canceler as shown in Fig.

\[
s_{out}(t) = s(t) - 2s(t + T) + s(t + 2T)
\]

Double-delay-line canceler

Three pulse canceler
- The output of the two single-delay line canceler in cascade is the square of that from a single canceler.  \[ H(f)_{DDL} = (H(f)_{SDLC})^2 = 4 \sin^2(\pi f_d T) \]
- The double delay line canceler and three pulse canceller are same frequency response function
- The weights of double delay line canceler or three pulse canceller are +1, -2, +1.
\[ s_{out}(t) = s(t) - 2s(t+T) + s(t+2T) \]
General form of a transversal (or nonrecursive) filter for MTI signal processing. This is known as transversal filter. [also sometimes known as a feedforward filter, a nonrecursive filter, a finite memory filter or a tapped delay-line filter.]
Canonical-configuration comb filter
Amplitude responses for three MTI delay-line cancelers.

(1) Classical three-pulse canceler (2) five-pulse delay-line canceler with "optimum" weights, and (3) 15-pulse Chebyshev design
Echoes from trees, vegetation, sea, rain, and chaff fluctuate with time, and these fluctuations can limit the performance of MTI radar. The experimentally measured power spectra of clutter signals may be approximated by

\[ W(f) = |g(f)|^2 = |g_0|^2 \exp \left[ -a \left( \frac{f}{f_0} \right)^2 \right] \]
Where, $W(f) = \text{clutter-power spectrum as a function of frequency}$

$g(f) = \text{Fourier transform of input waveform (clutter echo)}$

$f_0 = \text{radar carrier frequency}$

$a = \text{a parameter dependent upon clutter}$

Power spectra of various clutter targets.

1. Heavily wooded hills, 20 mi/h wind blowing ($a = 2.3 \times 10^{17}$);
2. Sparsely wooded hills, calm day ($a = 3.9 \times 10^{19}$);
3. Sea echo, windy day ($a = 1.41 \times 10^{16}$),
4. Rain clouds ($a = 2.8 \times 10^{15}$);
The clutter spectrum can also be expressed in terms of an \textit{rms} clutter frequency spread $\sigma_c$ in hertz or by the \textit{rms} velocity spread $\sigma_v$, in m/s. Thus,

$$W(f) = W_0 \exp \left( -\frac{f^2}{2\sigma_c^2} \right) = W_0 \exp \left( -\frac{f^2 \lambda^2}{8\sigma_v^2} \right)$$

where $W_0 = |g_0|^2$, $\sigma_c = 2\sigma_v / \lambda$, $\lambda$ = wavelength = $c/f_0$, and $c$ = velocity of propagation. It can be seen that $a = c^2/8\sigma_v^2$. The \textit{rms} velocity spread $\sigma_v$ is usually the preferred method for describing the clutter fluctuation spectrum.
The clutter attenuation is

\[ CA = \frac{\int_0^\infty W(f) \, df}{\int_0^\infty W(f) |H(f)|^2 \, df} \]

Where, \( H(f) \) is the frequency response function of the canceler
MTI Improvement Factor

The improvement factor can be written as

\[ I = \left( \frac{S_0}{S_i} \right)_{\text{ave}} \times \frac{C_i}{C_0} = \left( \frac{S_0}{S_i} \right)_{\text{ave}} \times \text{CA} \]

where \( S_0/C_0 \), output signal-to-clutter ratio,

\( S_i/C_i \), input signal-to-clutter ratio, and

\( \text{CA} = \) clutter attenuation.

The average is taken over all target doppler frequencies of interest.
MTI Improvement Factor for a single DLC

\[ CA = \frac{\int_0^\infty W(f) \, df}{\int_0^\infty W(f) |H(f)|^2 \, df} = \frac{\int_0^\infty W_0 \exp\left(-\frac{f^2}{2\sigma_c^2}\right) \, df}{\int_0^\infty W_0 \exp\left(-\frac{f^2}{2\sigma_c^2}\right) 4 \sin^2 \pi f T \, df} = \frac{0.5}{1 - \exp\left(-\frac{2\pi^2 T^2 \sigma_c^2}{\sigma_c^2}\right)} \]

\[ e^x = \frac{x^n}{n!} = 1 + \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{6} + \ldots \]

\[ CA = \frac{f_p^2}{4\pi^2 \sigma_c^2} = \frac{f_p^2 \lambda^2}{16\pi^2 \sigma_v^2} = \frac{af_p^2}{2\pi^2 f_0^2} \]

Where, \( f_p \) is the pulse repetition frequency = 1/T.

The average gain \((S_0/S_i)_{avg}\) of the single delay-line canceler can be equal to 2. Therefore. The Improvement factor is

\[ I_{1c} = \frac{f_p^2}{2\pi^2 \sigma_c^2} = \frac{f_p^2 \lambda^2}{8\pi^2 \sigma_v^2} = \frac{af_p^2}{\pi^2 f_0^2} \]

\[ \left(\frac{S_0}{S_i}\right)_{ave} \times CA \]
Find the improvement factor for a double delay line canceler, whose average gain is 6.

\[ I_{2c} \approx \frac{f_p^2}{8\pi^4 \sigma_c^4} = \frac{f_p^4 \lambda^4}{128\pi^4 \sigma_v^4} = \frac{a^2 f_p^4}{2\pi^4 f_0^4} \]
Non-Coherent MTI

Non-Coherent MTI

Diagram showing the components of a Non-Coherent MTI system, including a duplexer, transmitter oscillator (i.e., magnetron), modulator, local oscillator, IF amplifier, amplitude detector, and A-scope output.
Coherent Pulsed MTI Search Radar

Pulsed Coherent MTI Search Radar