Taylor–Green Vortex

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The Taylor–Green vortex is an exact closed form solution of 2-dimensional, incompressible Navier–Stokes equations. This 2-dimensional decaying vortex defined in the square domain, $0 \leq x, y \leq \pi$, serves as a benchmark problem for testing and validation of incompressible Navier–Stokes codes.

The 2-dimensional, incompressible Navier–Stokes system of equation (consisting of continuity and two momentum equations) written in Cartesian coordinate system is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

The Taylor–Green Vortex solution is given by

$$u = \sin x \cos y \, F(t)$$

$$v = -\cos x \sin y \, F(t)$$

where $F(t) = e^{-2\nu t}$, ν being the kinematic viscosity of the fluid. The pressure p can be obtained by substituting the velocity solution in the momentum equations and is given by

$$p = \frac{\rho}{4}\left(\cos 2x + \sin 2y\right) F^2(t)$$

For the purpose of solving the Navier–Stokes equation numerically the following velocity conditions can be used:

$$u(x, 0) = \sin x \, F(t)$$

$$u(x, \pi) = -\sin x \, F(t)$$

$$u(0, y) = 0$$

$$u(\pi, y) = 0$$

and

$$
\begin{aligned}
v(x,0) &= 0 \\
v(x,\pi) &= 0 \\
v(0,y) &= -\sin y F(t) \\
v(\pi,y) &= \sin y F(t)
\end{aligned}
$$

The streamline pattern and the velocity vector plot of the Taylor–Green vortex, at time $t = 0$, are shown below.